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Tube Flow of Non-Newtonian Polymer Solutions: Part II. Turbulent Flow

DONALD M. METER

University of Wisconsin, Madison, Wisconsin

A correlation of turbulent tube-flow friction factors for non-Newtonian polymer solutions, based upon the fluid property $\tau_{1/2}$ defined in Part I, has been found to represent data taken on seven solutions of Natrosol hydroxyethylcellulose in ½- and 1-in. I.D. smooth tubes, with an average accuracy of 10.8%. Also, for these seven solutions, this correlation gives somewhat more accurate predictions of the point of transition into turbulent flow than are made by the Ryan-Johnson stability theory.

The correlation must be considered only provisional, however, because it does not simplify to the limiting case of extremely dilute solutions. Plausible extensions based on models of steady-flow non-Newtonian viscosity behavior, and other possible correlation schemes utilizing viscoelastic fluid properties, are briefly discussed.

The importance of the viscoelastic properties of fluids in determining their turbulent flow behavior has been suggested by Atkinson (*1*), Ward (*22*), and more recently by Dodge and Metzner (*4*). Recent studies (*10, 14*) have indicated, however, that for many polymer solutions the normal stresses in the two directions perpendicular to the direction of flow are equal, and furthermore (*12, 23*) that for such fluids a definite correspondence exists between viscoelastic and steady flow non-Newtonian viscosity properties. The primary purpose of this paper is to suggest ways in which non-Newtonian viscosity information may be used to correlate turbulent friction factor data for the tube flow of viscoelastic polymer solutions. It is felt that this approach is of value, even though it is not appropriate for extension to other types of non-Newtonian fluids, such as soft gels and suspensions.

DIMENSIONAL ANALYSIS OF TUBE FLOW FOR PSEUDOPLASTIC POLYMER SOLUTIONS

A four-constant model of pseudoplastic behavior, applicable to polymer solutions, has been proposed in Part I (*13*). There the integration of this model for laminar flow in circular tubes shows that the friction factor f is dependent upon the following four dimensionless groups, which include the four model parameters:

$$f = f\left(\frac{D^2 \tau_m \rho}{\eta_0^2}, \frac{\tau_w}{\tau_m}, \alpha, \frac{\eta_\infty}{\eta_0}\right) \tag{1}$$

Donald M. Meter is with the Squibb Institute for Medical Research, New Brunswick, New Jersey.

It is to be expected that the same dimensionless groups will be important in turbulent flow, though, of course, the exact relationship between the groups will be different from that in laminar flow.

None of the groups in Equation (1) is a Reynolds number. A Reynolds number based upon this model may be defined as the group $(D <v_z> \rho)$ divided by the tube-apparent viscosity, or ratio $(\tau_w)/(8 <v_z>/D)$ of laminar-flow consistency variables; it is

$$N_{Re} = \frac{D <v_z> \rho}{\eta_0} \left\{ \left[1 + \frac{4}{\alpha+3}\left(\frac{\tau_w}{\tau_m}\right)^{\alpha-1} \right] - \left(\frac{\eta_\infty}{\eta_0}\right)\left(\frac{\tau_w}{\tau_m}\right)^{\alpha-1} \left[\frac{4}{\alpha+3} + \frac{2}{\alpha+1}\left(\frac{\tau_w}{\tau_m}\right)^{\alpha-1} \right] + \ldots \right\} \tag{2}$$

Note that this definition forces evaluation of the tube-apparent viscosity at the prevailing wall shearing stress, whether the flow is laminar or turbulent. If N_{Re} above were to be used, only three other dimensionless groups would be needed to specify f; probably α, τ_w/τ_m, and η_∞/η_0 would be chosen.

A priori, it would be expected that the group η_∞/η_0 would not be important except at extremely high shear stresses where the upper limiting viscosity of the fluid is being approached.

The Reynolds number of Equation (2) yields the relation $f = 16/N_{Re}$ for the laminar region; for turbulent flow an f vs. N_{Re} relation would require the introduction of additional dimensionless groups as indicated above. Investigations (*4, 12*) with a Reynolds number, along with

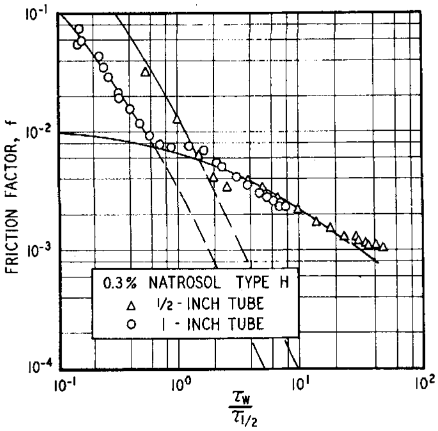

Fig. 1. Friction factor vs. $\tau_w/\tau_{1/2}$ for 0.3% Natrosol type H. $\tau_{1/2}$ = 92 dynes/sq.cm. Heavy solid line: f vs. $\tau_w/\tau_{1/2}$ correlation, Table 1. Light solid line: experimental laminar flow relation, Equation (4). Dashed line: the same, extended into the region where the flow actually is turbulent. The data points shown represent average velocities from 1.86 to 97.0 ft./sec. in the ½-in. tube, and from 0.644 to 26.2 ft./sec. in the 1-in. tube.

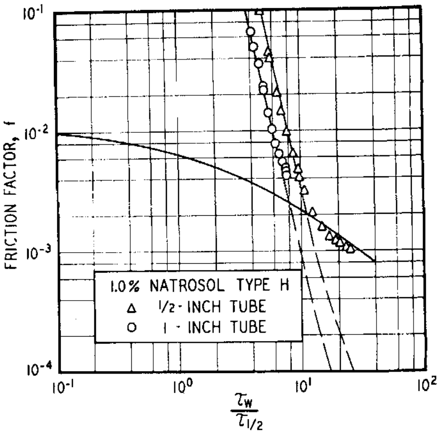

Fig. 2. Friction factor vs. $\tau_w/\tau_{1/2}$ for 1.0% Natrosol type H. $\tau_{1/2}$ = 173 dynes/sq.cm. Heavy solid line: f vs. $\tau_w/\tau_{1/2}$ correlation, Table 1. Light solid line, experimental laminar flow relation, Equation (4). Dashed line: the same, extended into the region where the flow actually is turbulent. The data points shown represent average velocities from 4.27 to 95.0 ft./sec. in the ½-in. tube, and from 4.84 to 26.1 ft./sec. in the 1-in. tube.

other dimensionless groups based on non-Newtonian viscosity properties, have not led to a reliable, convenient correlation of turbulent flow data for viscoelastic polymer solutions. Therefore, attention has reverted to the use of dimensionless groups such as those of Equation (1).

EXPERIMENTAL DATA

Pressure drop vs. flow rate data have been taken in circular, smooth-walled ½- and 1-in. I.D. brass tubes. Flow rates from 1 to 70 gal./min. were attainable; flow rate measurement was done by weighing the fluid collected in a given time. Pressure drops were measured by water-over-mercury and water-over-oil manometers connected to various combinations of pressure taps 100, 150, 175, and 200 diameters downstream from the test section entrance; the longest span possible between taps, without blowing the manometers, was used.

The pipe flow data encompassed both laminar and turbulent flow regimes. Laminar flow viscometric data at high shear stresses were obtained from a capillary-tube viscometer, and values of the lower limiting viscosity were determined by measuring the fall rate of small glass beads. More refined methods of η_0 measurement are now available (*18, 21*).

Fluids studied included seven aqueous solutions of Natrosol 250 hydroxyethylcellulose, namely, 0.3, 0.5, 0.7, and 1.0% Natrosol type H (high viscosity grade) and 1.0, 1.5, and 2.0% type G (special medium viscosity grade), all at 77°F.

Further details concerning the experimental apparatus and results are available elsewhere (*12*).

CORRELATION OF TURBULENT FLOW FRICTION FACTORS FOR NATROSOL SOLUTIONS

Values of the model parameters η_0, η_∞, α, and τ_m and also of the fluid property $\tau_{1/2}$, were determined as outlined in Part I (*13*) for all seven Natrosol solutions. Values of the lower limiting viscosity η_0 were subject to some error both due to experimental techniques and to fluid aging; where it was deemed necessary, they were adjusted to agree with extrapolated laminar pipe flow data. These adjusted values were then used to specify the model parameter η_0 and also the fluid property, $\tau_{1/2}$.

It has been found, for both pipe diameters and for all seven Natrosol solutions, that data points in the turbulent regime give a substantially single-valued relation when expressed in terms of f and τ_w/τ_m. Since these solutions were sufficiently concentrated that $\tau_m \approx \tau_{1/2}$, a single-valued relationship between f and $\tau_w/\tau_{1/2}$ also has been found. The other dimensionless groups of Equation (1) do not appear to affect this correlation; in this study values of $D^2\tau_m\rho/\eta_0^2$ varied from 0.675 to 64,000, and values of α ranged from 1.96 to 3.10, with no detectable influence. This is an extremely surprising result.

The determination of $\tau_{1/2}$ from its definition is straightforward (though to determine it η_0 must be known precisely), whereas the determination of τ_m involves the arbitrariness inherent in model fitting. For this reason attention is here focused on a $\tau_w/\tau_{1/2}$ correlation of friction factor data; a possible eventual need to use τ_w/τ_m will be discussed later.

A smoothed curve of f vs. $\tau_w/\tau_{1/2}$ has been prepared from the turbulent data on these seven Natrosol solutions; selected points from this smoothed curve are listed in Table 1. This curve may be approximated over a sizeable range $(0.2 < \tau_w/\tau_{1/2} < 10)$ by

$$f \cong 0.0064 - 0.00425 \log_{10} (\tau_w/\tau_{1/2}) \tag{3a}$$

and at higher ranges $(10 < \tau_w/\tau_{1/2} < 40)$ by

TABLE 1. $\tau_w/\tau_{1/2}$ TURBULENT FLOW FRICTION FACTOR CORRELATION FOR NATROSOL SOLUTIONS

$\tau_w/\tau_{1/2}$	f
0.1	0.0098
0.2	0.0092
0.3	0.0087
0.4	0.0082
0.6	0.0074
0.8	0.0068
1.0	0.0064
1.5	0.0056
2.0	0.0051
3.0	0.0043
4.0	0.00378
6.0	0.00300
8.0	0.00250
10.0	0.00215
15.0	0.00162
20.0	0.00132
30.0	0.00098
40.0	0.00080

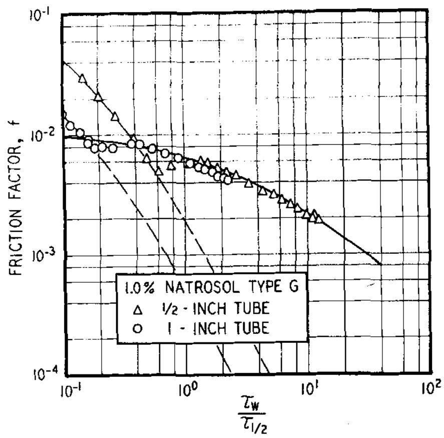

Fig. 3. Friction factor vs. $\tau_w/\tau_{1/2}$ for 1.0% Natrosol type G. $\tau_{1/2}$ = 580 dynes/sq.cm. Heavy solid line: *f* vs. $\tau_w/\tau_{1/2}$ correlation, Table 1. Light solid line, experimental laminar flow relation, Equation (4). Dashed line: the same, extended into the region where the flow actually is turbulent. The data points shown represent average velocities from 2.51 to 90.6 ft./sec. in the ½-in. tube, and from 2.87 to 26.0 ft./sec. in the 1-in. tube.

$$f \cong 0.0117\ (\tau_w/\tau_{1/2})^{-0.73} \tag{3b}$$

The curve of Table 1 is shown in Figures 1 to 4, together with data on four of the seven Natrosol solutions.

The laminar flow friction factors shown in these figures are calculated from laminar flow consistency variable ($8 <v_z>/D$ vs. τ_w) data by the equation

$$f = \frac{2\tau_w}{\rho <v_z>^2} = \frac{128\,\tau_w}{D^2\rho\left(\dfrac{8<v_z>}{D}\right)^2} \tag{4}$$

Values of the group $\tau_w/\tau_{1/2}$ also are evaluated at the wall shear stress τ_w. To the extent that the model discussed in Part I (*13*) represents actual fluid behavior, Equation (8) of that paper may be used in place of Equation (4) above. In the case of the present fluids, laminar flow friction factors thus calculated from the model superimpose exactly upon those calculated from Equation (4), over virtually the entire shear stress range shown in Figures 1 to 4.

The intersection of these laminar curves with the turbulent flow f vs. $\tau_w/\tau_{1/2}$ relation of Table 1 allows prediction of the friction factor at the transition between laminar and turbulent flow. As seen in Figures 1 to 4, especially in the case of the ½-in. tube, the actual point of abrupt transition, when an abrupt transition does occur, is often at higher wall shear stress than this intersection predicts.

In the turbulent regime, beyond the transition points predicted above, the average absolute deviation of 149 friction factors calculated from the correlation of Table 1, from experimental friction factors obtained at the same $\tau_w/\tau_{1/2}$ for the above seven Natrosol solutions in ½- and 1-in. tubes, is 10.8%. Comparable agreement has been found for data on 0.3 and 0.7% carboxymethylcellulose (CMC) solutions taken by Dodge (*3*).

The $\tau_w/\tau_{1/2}$ correlation, being based upon friction factor data taken on viscoelastic Natrosol solutions, reflects the damping of turbulent fluctuations and therefore the low turbulent flow friction factor values (down to 0.001 or even less) which these fluids exhibit. In comparison, friction factors lower than 0.004 are very rarely encountered in the flow of suspensions, soft gels (such as Carbopol), or Newtonian fluids.

PROCEDURE FOR USE OF THE CORRELATION

The use of this correlation for the flow of a given fluid (with known $\tau_{1/2}$, density ρ, and laminar flow $8<v_z>/D$ vs. τ_w relation) in a pipe of diameter D may be broken down into two steps: the determination of the predicted point of transition into turbulent flow and the determination of the turbulent flow $<v_z>$ vs. $\Delta p/L$ curve. First, a laminar flow plot of f vs. $(\tau_w/\tau_{1/2})$ is prepared from Equation (4) and the above known fluid and pipe properties. The intersection of this curve with that of the turbulent flow correlation (Table 1) gives the values of f and $(\tau_w/\tau_{1/2})$ at the predicted transition point. The average velocity $<v_z>$ and pressure gradient $\Delta p/L$ at this point are simply

$$<v_z> = \sqrt{\frac{2(\tau_w/\tau_{1/2})\tau_{1/2}}{\rho f}} \tag{5}$$

$$\frac{\Delta p}{L} = \frac{4(\tau_w/\tau_{1/2})\tau_{1/2}}{D} - \rho g \tag{6}$$

where g is the axial component of gravitational acceleration. Second, beyond the transition point, pairs of f vs. $\tau_w/\tau_{1/2}$ values from Table 1 may be inserted into Equations (5) and (6) to yield a turbulent flow curve of $<v_z>$ vs. $\Delta p/L$.

COMPARISON WITH PREVIOUS CORRELATIONS

Ryan and Johnson (*15*) have proposed a stability parameter to predict the transition point between laminar and turbulent pipe flow; their treatment has been generalized by Hanks (*8, 9*). The critical friction factors which their treatment predicts, when used with the Ostwald-de Waele power model, range from 0.00762 (for a Newtonian fluid, n = 1.0) down only to 0.00668 (at n = 0.416). When a model (such as the Ellis model) incorporating a lower limiting viscosity is used, critical friction factors in the same range are predicted (*12*). The experimental Natrosol friction factors at the point of abrupt transition, however, sometimes are considerably lower than the Ryan-Johnson predicted values (see especially Figures 2 and 4) owing to the damping of turbulent motion which occurs in viscoelastic fluid systems. Transition points predicted by the intersection of laminar data with

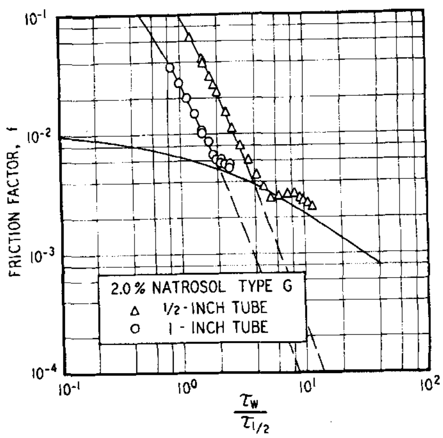

Fig. 4. Friction factor vs. $\tau_w/\tau_{1/2}$ for 2.0% Natrosol type G. $\tau_{1/2}$ = 650 dynes/sq.cm. Heavy solid line: *f* vs. $\tau_w/\tau_{1/2}$ correlation, Table 1. Light solid line, experimental laminar flow relation, Equation (4). Dashed line: the same, extended into the region where the flow actually is turbulent. The data points shown represent average velocities from 5.14 to 81.6 ft./sec. in the ½-in. tube, and from 5.71 to 26.0 ft./sec. in the 1-in. tube.

the f vs. $(\tau_w/\tau_{1/2})$ correlation represent this retarded turbulence phenomenon somewhat better than do those predicted by the Ryan-Johnson treatment, especially in the case of the ½-in. tube.

The accuracy (10.8%) of the $\tau_w/\tau_{1/2}$ correlation of Table 1 in representing the present Natrosol data is very slightly better than that (12%) claimed by Shaver (*16*) in using the Shaver-Merrill correlation (*17*) to fit his data on similar CMC and ammonium alginate solutions. It is a significant improvement in accuracy, however, over the ability of the Shaver-Merrill correlation to describe the present Natrosol data (*12*).

The correlations of Dodge and Metzner (*4*), of Thomas (*19, 20*), and of Clapp (*2*) are based upon data taken on soft gels and suspensions. A yield stress is sometimes required to characterize the zero shear non-Newtonian viscosity behavior of these fluids (*6, 7, 19*), so that a correlation based upon the existence of a lower limiting viscosity is inappropriate for use with such materials. Likewise, the Dodge-Metzner, Thomas, and Clapp correlations do not provide for the low friction factors found with Natrosol, CMC, and similar polymer solutions.

OTHER CORRELATION POSSIBILITIES

Since $\tau_{1/2}$ (the shear stress at which $\eta = \eta_0/2$) loses its meaning in the limiting case of extremely dilute polymer solutions, it is to be expected that the $\tau_w/\tau_{1/2}$ correlation of Table 1 will not hold for these fluids. This limitation must be overcome if a dimensional analysis approach in the manner of Equation (1) is to be generally useful. Since a limiting value of τ_m at infinite dilution for each polymer should exist (as long as η_0 and η_∞ are not equal, there will be a shear stress τ_m at which η is the average of η_0 and η_∞), the group τ_w/τ_m appears to be more appropriate than $\tau_w/\tau_{1/2}$ for use as the basis of a comprehensive correlation. In addition, perhaps other, or all, of the dimensionless groups in Equation (1) would be needed to allow representation of data on both dilute and concentrated polymer solutions. It is to be hoped that this approach will be tested further, with data taken on solutions of many polymers, over large ranges of polymer concentration, flow rate, and pipe diameter.

Dodge and Metzner (*4*) have shown that non-Newtonian viscosity data alone are not capable of yielding a turbulent flow correlation which treats both polymer solutions and also soft gels and suspensions. Recent attempts by McEachern and the author [reported in (*12*)] to interpret these differences in terms of the dynamic shear modulus G' of linear viscoelasticity (*5*) have not been successful. There are indications (*11, 12*), however, that parameters based upon measures of normal stress behavior might allow interpretation of the turbulent flow behavior of many types of non-Newtonian fluids. A definitive test of this latter approach must await the ability to measure very slight normal stresses in various types of fluids, at appreciable rates of shear.

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NOTATION

D = diameter of tube
f = friction factor
g = axial component of gravitational acceleration
G' = dynamic shear modulus
L = length of tube
n = exponent parameter in power law ($= 1/\alpha$)
N_{Re} = Reynolds number
p = pressure
$\langle v_z \rangle$ = average velocity in tube flow

Greek Letters

α = parameter in several rheological models
Δ = difference in
η = non-Newtonian viscosity
η_0 = lower limiting viscosity
η_∞ = upper limiting viscosity
ρ = fluid density
τ_m = parameter in Equation (2) of Part I (*13*)
$\tau_{1/2}$ = parameter in Equation (4) of Part I (*13*)
τ_w = shear stress at tube wall

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